Spectral Signatures of Active Fluctuations in Semiflexible Polymers

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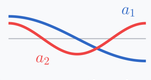
Take-home: An active bath does not heat a polymer uniformly. Instead, it **redistributes fluctuations across modes**, so the polymer's mode spectrum resolves how nonequilibrium forcing is spread over length and time scales.

1 Background: a filament as a multiscale thermometer

In equilibrium a *single* temperature governs every mode; active matter breaks this [3]. We build on the classic shape–fluctuation method [1,2] that turns a fluctuating filament into a thermometer, and extend it *mode by mode*.

Image a fluctuating filament (microtubules, actin)
Gittes [1]; Brangwynne [2]

⇒



Decompose the shape into bending modes
$u_\perp(s) = \sum_n a_n \phi_n(s)$

⇒

$$\langle a_n^2 \rangle_{\rm eq} = \frac{k_B T}{\kappa\, q_n^2}$$

fit ⇒ κ
Equipartition fixes the bending rigidity κ
(same κ we set in sims)

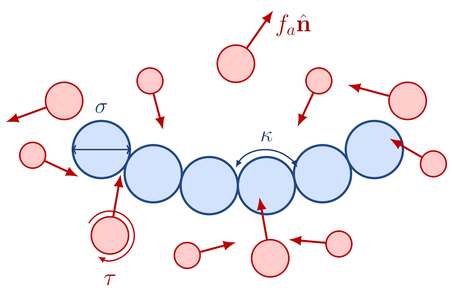
⇒

$$k_B T_n^{\rm eff} \equiv \kappa\, q_n^2 \langle a_n^2 \rangle$$

activity ⇒ $T_n^{\rm eff} \neq T$
Out of equilibrium the same relation **defines** a
per-mode temperature

2 Model: a polymer in an active bath

Semiflexible bead–spring chain: harmonic bonds, cosine bending stiffness κ, WCA excluded volume. Two realizations of activity:



- **Semiflexible polymer** beads of diameter σ, bending κ
- **Active bath** of ABPs
- → Self-propulsion $f_a\hat{\mathbf{n}}$ in *random* directions
- ⟳ Reorientation; persistence $\tau = 1/D_r$

Schematic of a semiflexible chain (bending rigidity κ) in a noisy active bath; arrows show random self-propulsion $f_a\hat{\mathbf{n}}$. The drawing is illustrative; the simulated bath is monodisperse (equal-size particles, identical self-propulsion force).

Polymer beads are *passive*; activity enters only through the conservative force $\mathbf{F}_i^c$ (bonds, bending, excluded volume):

$$m\dot{\mathbf{v}}_i = -\gamma_t \mathbf{v}_i + \mathbf{F}_i^c + \sqrt{2\gamma_t k_B T}\,\boldsymbol{\xi}_i$$

(i) Explicit ABP bath. Self-propelled particles (f_a, persistence τ) collide with the chain:

$$m_a\dot{\mathbf{V}}_j = -\gamma_t \mathbf{V}_j + \mathbf{F}_j^c + f_a\hat{\mathbf{n}}_j + \sqrt{2\gamma_t k_B T}\,\boldsymbol{\Xi}_j$$

(ii) Implicit OU bath. A colored force $\mathbf{F}_i^{\rm OU}$ is added to each bead:

$$\langle \mathbf{F}_i^{\rm OU}(t)\cdot\mathbf{F}_j^{\rm OU}(t')\rangle = (f_a^{\rm OU})^2\, e^{-|t-t'|/\tau^{\rm OU}}\,\delta_{ij}$$

Simulated in 2D with LAMMPS (underdamped Langevin dynamics, reduced LJ units); two bending stiffnesses κ, with the active force f_a and persistence τ swept over a broad range.

3 Theory: explicit bath → effective colored noise

Active force density on the contour, $\mathbf{F}_{\rm act}(s,t) = \sum_i \mathbf{f}\big(\mathbf{R}_i(t) - \mathbf{r}(s,t)\big)$.

A single-particle propagator + Gaussian-displacement closure, projected onto tangent modes, gives a mode-diagonal active kernel:

$$\langle \xi_n^{\rm act}(t)\,\xi_m^{\rm act}(0)\rangle = \delta_{nm}\, q_n^2 D_0\, e^{-q_n^2\sigma^2}\, e^{-|t|/\tau_{\rm eff}}$$

Assumptions: dilute bath of freely-moving ABPs; nearly straight (weak-bending) chain; one effective memory time $\tau_{\rm eff}$ (fitted, not the simulation input τ).

Overdamped mode dynamics ⇒ steady-state mode variance:

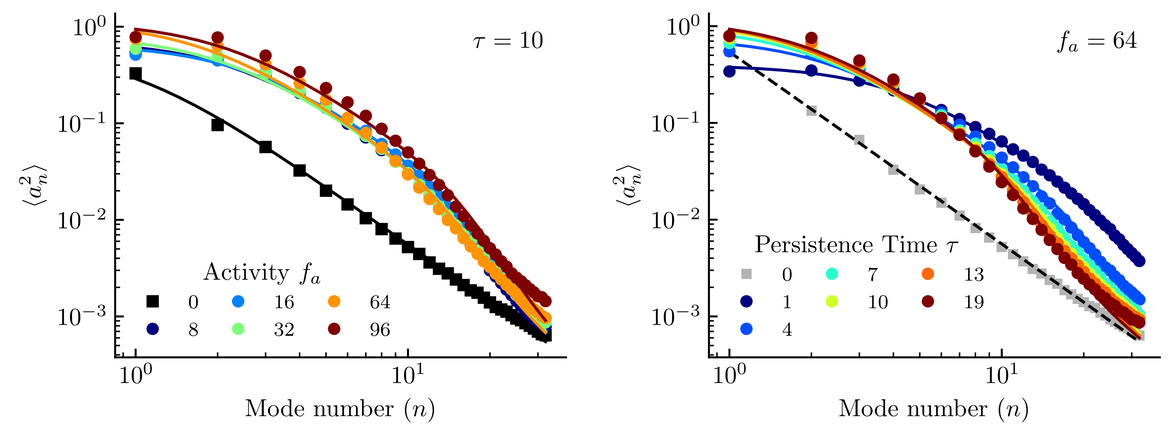
$$\langle a_n^2\rangle = \frac{1}{\kappa q_n^2}\left[k_B T + \frac{D_0\tau_{\rm eff}}{\zeta\,(1+\tau_{\rm eff}\omega_n)}\, e^{-q_n^2\sigma^2}\right],\quad \omega_n = \frac{\kappa q_n^4}{\zeta}$$

Symbols: s contour coordinate, L contour length; n mode number, $q_n = n\pi/L$ mode wavenumber; a_n mode amplitude, $\xi_n^{\rm act}$ active force on mode n; κ bending rigidity, $\zeta \approx \gamma_t$ monomer friction; $\omega_n = \kappa q_n^4/\zeta$ mode relaxation rate; $k_B T$ thermal energy (T bath temperature), $T_n^{\rm eff}$ per-mode effective temperature. *Effective active-bath parameters (fit to the spectra):* D_0 forcing strength; $\tau_{\rm eff}$ memory (persistence) time; σ spatial range of the force.

Equivalently (panel 1), each mode carries its own $T_n^{\rm eff} = \kappa q_n^2\langle a_n^2\rangle/k_B$.

- Low modes ($\tau_{\rm eff}\omega_n \ll 1$) respond coherently to the bath and are strongly enhanced.
- High modes: the active term falls off as $\sim n^{-6}$ (the $1/\kappa q_n^2$ prefactor × the $1+\tau_{\rm eff}\omega_n \propto n^4$ denominator) vs. the thermal $\sim n^{-2}$, so its relative weight dies as n^{-4} and high modes stay near-passive. ($e^{-q_n^2\sigma^2}$ adds further suppression, weak over the fitted modes.)
- ⇒ each mode runs at its **own effective temperature** rather than one shared T.

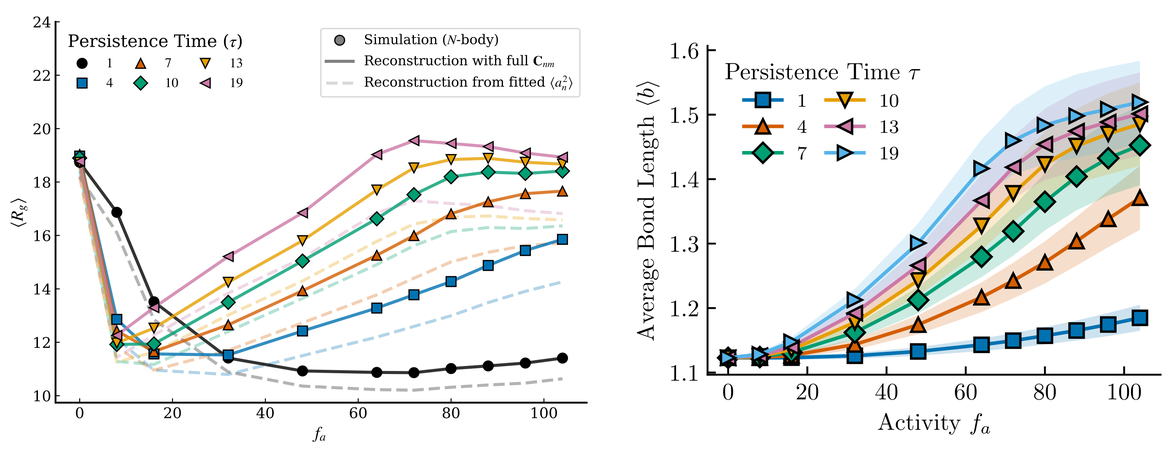
4 Active force & persistence time reshape the mode fluctuations



Mode variance $\langle a_n^2\rangle$ vs. mode n. Symbols: simulation; solid lines: theory (fit); dashed: passive ($f_a = 0$) reference. Left: fixed τ, increasing f_a. Right: fixed f_a, increasing τ. ($\kappa = 32\,k_B T$.)

- Increasing f_a enhances **mainly the lowest modes**; high modes stay near-passive, so the drive is selective.
- Increasing τ shifts weight to lower mode number: persistence selects which modes are driven.
- The crossover between activity-dominated and near-passive sectors moves to lower n as f_a or τ grows.
- Fitted theory reproduces both magnitude and crossover across the sweep.

5 Polymer swelling and the role of bond stretching

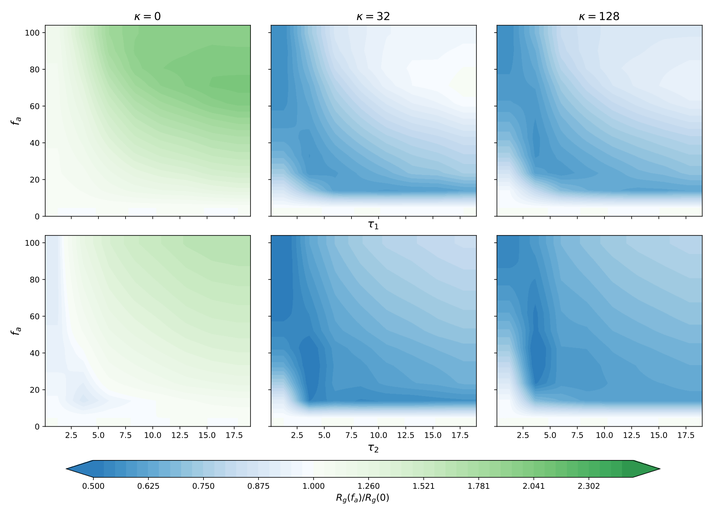


Left: radius of gyration $\langle R_g\rangle$ vs. f_a. Right: mean bond length $\langle b\rangle$ and its spread grow with activity (stronger at large τ), nearly independent of κ.

- Diagonal-mode theory ($\langle b_n\cdot b_m\rangle \propto \delta_{nm}$) gets the swelling trend but **underestimates** R_g at large f_a, τ.
- The exact finite-N reconstruction from the *full* $\mathbf{C}_{nm}$ recovers R_g, so the shortfall is physics beyond the diagonal modes: bond stretching (contour-length renormalization) and omitted cross-mode terms $\langle \mathbf{b}_n\cdot\mathbf{b}_m\rangle_{n\neq m}$.
- Mean bond length $\langle b\rangle$ grows with f_a, τ (right panel); the fixed-contour theory misses this. **Work in progress** with extensible bonds.

$\mathbf{b}_n$: discrete bond-mode amplitudes (finite-N counterpart of the tangent modes a_n); $\mathbf{C}_{nm} = \langle \mathbf{b}_n\cdot\mathbf{b}_m\rangle$ their covariance.

6 Explicit and implicit baths: same large-scale trends



Normalized $R_g(f_a)/R_g(0)$ (vertical axis f_a). Top row: explicit ABP bath, horizontal axis τ_1 = ABP persistence $\tau = 1/D_r$. Bottom row: implicit OU noise, horizontal axis τ_2 = OU persistence $\tau^{\rm OU}$. Columns: $\kappa = 0$ (flexible limit), 32, 128.

- Explicit and implicit baths give very similar swelling landscapes across κ: colored noise captures the leading large-scale behaviour [4]. *(Qualitative: no one-to-one parameter mapping imposed.)*
- Finite particle size, excluded volume, crowding & bond stretching leave **signatures beyond** a minimal implicit model.

7 Conclusions & outlook

- Activity reorganizes fluctuations mode by mode: persistence vs. mode relaxation sets a **mode-dependent** $T_n^{\rm eff}$, captured by a reduced colored-noise theory with a few effective parameters.
- Global size (R_g) needs physics beyond the fixed contour (bond stretching, cross-mode coupling); **work in progress** with extensible bonds.
- **Outlook:** the polymer's mode spectrum could help characterize active media (their effective time and length scales), as suggested by chromatin motion whose coupling to transcription depends on local compaction [5].

Selected references

This work: L. Grover, A. K. Dasanna & A. Chaudhuri, *Spectral Signatures of Active Fluctuations in Semiflexible Polymers*, arXiv:2604.10168 (2026).

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arXiv:2604.10168
Read the paper

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Acknowledgments: We thank IISER Mohali for computational resources and support.